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# Technological vs ecological switch and the environmental Kuznets curve

**RAOUF BOUCEKKINE, AUDE POMMERET, AND FABIEN PRIEUR**

For quite a long time, it has been claimed that the relationship between income and pollution was inverted U-shape, yielding the so-called Environmental Kuznets Curve (EKC hereafter). This claim, based on several early empirical studies (see for example, Grossman and Krueger 1993), has been at the heart of a massive empirical and theoretical literature. Empirical research has mainly consisted in examining a wide variety of pollutants for evidence of the inverted U-shaped pattern, resulting in the conclusion that such a shape is valid for many local and flow pollutants. But it does not seem to be the rule for stock pollutants like  $\text{CO}_2$  which do rather generate monotonically increasing relation between wealth and pollution (see Brock and Taylor 2005, for a survey). Parallely, a great deal of theoretical contributions has been devoted to identify conditions under which the EKC arises. This includes optimal growth models like in Dinda (2004) and Stokey (1998) or equilibrium models in the spirit of John and Pecchenino (1994), all with stock pollutant. In all the contributions mentioned above, the role of the abatement technology is crucial. For example, in the latter abatement starts only after a large amount of capital, and thus pollution, is accumulated, ultimately generating the decreasing part of the EKC. A similar scheme can give rise to the EKC under endogenous growth: the shift from insufficient to sufficient investment in abatement in an advanced stage of development curves down the pollution level at that stage. Another stream of the EKC literature puts forward technological progress: if richer countries are supposed to use, say, more energy-saving technologies,

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which are typically costly to implement, then pollution goes down in sufficiently developed countries. We shall take this avenue in this paper. Precisely, we consider an optimal technology adoption AK model in line with Boucek-Kine, Krawczyk and Vallée (2011): an economy, caring about consumption and pollution as well, starts with a given technological regime and may decide to switch at any moment to a cleaner technology at a given permanent or transitory output cost. At the same time, we posit that there exists a pollution threshold above which the assimilation capacity of Nature goes down, featuring a kind of irreversible ecological regime. It has been shown by Prieur (2009) that introducing such an irreversibility in the John and Pecchenino model considerably weakens the EKC case. We shall study how the irreversibility mechanism interacts with the ingredients of the optimal technological switch problem outlined above, with a special attention to the outcomes regarding the capital-pollution relationship.

Our contributions are twofold. First of all, our contribution is technical. It is well known that including irreversibility in the above sense also induces an optimal switching time problem from the reversible to the irreversible ecological regime. Accordingly, our optimal AK growth model involves two optimal timing problems associated to technological and ecological switching times respectively. Original multi-stage optimal control techniques will be developed to solve the model, extending previous works of Tomiyama (1985) on technological switching and Tahvonen and Withagen (1996) on ecological switching. Second, and more importantly, the interaction between the ecological and technological mechanisms generates a large set of potential optimal solutions. These solutions feature different relationships between capital and pollution. For our calibrated model, if a single technological switch is optimal, one recovers the EKC provided initial pollution is high enough. If exceeding the ecological threshold is optimal, then the latter configuration is far from being the rule.

## The problem

We consider an optimal growth AK model with two stocks, physical capital ( $K$ ) and pollution ( $P$ ). Two production technologies are available from  $t = 0$ . Each can be described by two parameters  $(A_i, q_i)$ ,  $i = 1, 2$ :  $A_i$  is marginal productivity of capital since the production function is  $Y = A_i K$ , while  $q_i$  measures the degree of dirtiness of the technology  $i$ . Concretely,  $q_i$  captures the marginal contribution of capital to the flow of pollution. The economy starts with technology  $i = 1$  and has to decide whether it switches to technology  $i = 2$ , and **when**. The economy cares about both the levels of consumption and pollution. So for the problem to be nontrivial, we shall assume that  $A_1 > A_2$  but  $q_1 > q_2$ : technology 1 is more productive but dirtier.  $A_1 - A_2$  measures the cost of adopting a cleaner technology, we assume it permanent. The problem is at this stage similar to the one tackled in Boucekine Krawczyk and Vallée (2011). There are however two major differences with respect to the latter contribution: the existence of capital accumulation and of pollution decay. We also introduce a key feature: irreversible pollution as in Prieur, Tidball and Withagen (2011). More precisely, the two state variables evolve according to the following laws of motion:  $\dot{K} = (A_i - \delta)K - C$ , for  $i = 1, 2$ ,  $C$  being consumption and  $\delta$  the depreciation rate, and  $\dot{P} = q_i K - \alpha_j P$ , with  $i, j = 1, 2$  and  $\alpha_1 > \alpha_2 \geq 0$ . The  $j$  subscript indexes the ecological regime, here parameterized by the pollution natural decay rate  $\alpha_j$ . While the technological switch from regime  $i = 1$  to  $i = 2$  does not require any minimal level of physical capital to take place, an ecological switch from  $j = 1$  to  $j = 2$ , where  $\alpha_1 > \alpha_2 \geq 0$ , does naturally entail the idea that nature cannot regenerate in the same way for low and high levels of the stock of pollution. Typically, this is modeled through a threshold value for the pollution stock, say  $\bar{P}$ , above which the decay rate falls.<sup>1</sup>

As explained in the introduction section, putting together technological and ecological switches enriches considerably the economic discussion. From the technical point of view, the problem sounds at first glance strongly asymmetric, the technological switch involving

explicit timing decisions while ecological switches are essentially based on a threshold pollution level posited. It is however easy to see that this apparent asymmetry can be attenuated: in line with the pioneering contribution of Tahvonen and Withagen (1996), it is quite obvious to reformulate the ecological switching problem also as an optimal timing problem: indeed, if such a switch occurs, it exists a date, say  $t_P$ , where  $P_{t_P} = \bar{P}$  (assuming  $P_0 < \bar{P}$ ). As we shall see, this does not mean that the two state variables' laws of motion will imply similar optimality conditions. However, the previous observation legitimates the formulation of the two switchings problems both as optimal timing problems. Hereafter,  $t_K$  with  $0 \leq t_K \leq \infty$  will refer to the timing of technological switching while  $t_P$  with  $0 < t_P \leq \infty$  does the job on the ecological side.

Given these provisos, our optimal control problem can be written as:

$$\max_{\{C, t_K, t_P\}} V = \int_0^{\infty} [U(C) - D(P)] e^{-\rho t} dt$$

subject to,

$$\dot{K} = \begin{cases} (A_1 - \delta)K - C & \text{if } t \leq t_K \\ (A_2 - \delta)K - C & \text{else} \end{cases}$$

and,

$$\dot{P} = \begin{cases} q_1 K - \alpha_1 P & \text{if } t \leq \min\{t_K, t_P\} \\ q_1 K - \alpha_2 P & \text{if } t_P < t \leq t_K \\ q_2 K - \alpha_1 P & \text{if } t_K < t \leq t_P \\ q_2 K - \alpha_2 P & \text{if } t > \max\{t_K, t_P\} \end{cases}$$

$P_0 < \bar{P}$  and  $K_0$  are given,  $\rho > 0$  is the rate of pure time preference. The control set includes the two timing variables mentioned above, plus consumption. The social welfare function is the same as in Tahvonen and Withagen (1996). The following standard regularity conditions assure the concavity of the problems we will have to handle along the way:

**Hypothesis 1.** *The utility function,  $U(C)$ , satisfies:  $U(0) = 0$ ,  $U''(C) < 0$ ,  $0 < U'(0) < \infty$  and  $\exists \tilde{C}/U'(\tilde{C}) = 0$ . The damage function,  $D(P)$ , satisfies:  $D(0) = 0$ ,  $D'(P) > 0$ ,  $D''(P) \geq 0$  and  $D'(0) = 0$ .*

The optimal control problem stated above is novel in that the two timing problems are of a different nature. In particular, one involves a threshold level for the state variable and the other no. Problems with multiple timing have already been tackled in the literature (see Saglam 2011) but in the latter literature only technological switching is considered. Here two types of timing problems are mixed in the same framework; needless to say, the interaction of both is very likely to give rise to a richer set of outcomes. Indeed, a quick inspection into the set of optimal regimes allowed in our enlarged problem is enough to get this point. A priori, one can list the following optimal regimes: 1. No switch, 2. One technological switch, 3. One ecological switch, 4. Two switches: technological then ecological and 5. Two switches: ecological then technological. It is not obvious at all to guess a priori which kind of regime will result optimal given initial conditions, preferences and available technological and ecological menus. Even worse, one can identify within the optimal outcomes with at least one switch (the last five categories), different classes of solutions: interior (that is  $t_K > 0$  and/or  $t_P > 0$ ) or corner ( $t_K = 0$ ). Eight regimes are thus possible. Even more, one might be interested in distinguishing the case when there is a simultaneous ecological and technological interior switch, which adds another possibly interesting sub-case.

The next section gives our solution approach to this intricate problem.

### **The solution approach**

We shall proceed as follows. First, for every possible regime  $k$  ( $k = 1$  to 10 regimes from the discussion just above), we write the corresponding first-order necessary conditions and compute the resulting welfare function, say  $V_k$ . Then, we pick the regime which delivers the largest social welfare, that is we identify the global maximum of the problem. The chal-

lenge is first analytical because the general optimal control problem involved is nontrivial. It is also computational because comparing nine possible regimes is highly demanding. We shall ultimately resort to numerical comparison because our underlying optimal control problem does not admit a closed-form solution in most regimes. In the following, we provide the general control theory foundations to identify the solutions with two interior switches; this covers three cases:  $0 < t_K < t_P < \infty$ ,  $0 < t_P < t_K < \infty$ , and  $0 < t_K = t_P < \infty$ . The other cases can be immediately recovered from the literature with one possible technological switch (see Boucekkine, Saglam and Vallée 2004) or one possible ecological switch (Tahvonen and Withagen, 1996), including corner regimes.<sup>2</sup>

A natural approach is to decompose the problem into several sub-problems for given timing variables, to solve each of them, and then to identify the optimal timings. With one timing variable, two sub-problems are involved corresponding to the resulting two time intervals, before and after the switch. In our case, three would result from the occurrence of two switching times. In the spirit of Tomiyama (1985), we shall use the following recursive scheme, illustrated here below on the case  $0 < t_K < t_P < \infty$ :

- *Third interval sub-problem*: the problem in this regime is:

$$\max_{\{C\}} V_3 = \int_{t_P}^{\infty} [U(C) - D(P)] e^{-\rho t} dt$$

subject to,

$$\begin{cases} \dot{K} = (A_2 - \delta) K - C \\ \dot{P} = q_2 K - \alpha_2 P \end{cases}$$

where  $t_P$  and the initial conditions  $K(t_P)$  and  $P(t_P) = \bar{P}$  are fixed. The associated hamiltonian is:  $H_3 = [U(C) - D(P)] e^{-\rho t} + \lambda_K^{22} ((A_2 - \delta) K - C) + \lambda_P^{22} (q_2 K - \alpha_2 P)$ , where  $\lambda_v^{ij}$  is the co-state variable associated with the state variable  $v = K, P$  in the technological menu  $i$  and ecological regime  $j$ . The resulting value-function is of the form  $V_3^*(t_P, K(t_P))$ .



- *Second interval sub-problem:* in the next interval, the maximization problem is:

$$\max_{\{C, t_P, K(t_P)\}} V_2 = \int_{t_K}^{t_P} [U(C) - D(P)] e^{-\rho t} dt + V_3^*(t_P, K(t_P))$$

subject to the corresponding dynamics, for regime  $i = 2$  and  $j = 1$ , where  $t_K$ ,  $K(t_K)$  and  $P(t_K)$  are given, and  $t_P$  and  $K(t_P)$  are free. Respectively denote by  $H_2$  and  $V_2^*(t_K, K(t_K), P(t_K))$  the corresponding hamiltonian and the resulting value-function.

- *First interval sub-problem:* This sub-problem considers the interval  $[0, t_K]$ :

$$\max_{\{C, t_K, K(t_K), P(t_K)\}} V_1 = V = \int_0^{t_K} [U(C) - D(P)] e^{-\rho t} dt + V_2^*(t_K, K(t_K), P(t_K))$$

subject to the dynamics of regimes  $i = 1$  and  $j = 1$ , with  $K(0)$  and  $P(0)$  given, and with free  $t_K$ ,  $K(t_K)$  and  $P(t_K)$ . Again, denote the hamiltonian by  $H_1$  and it is obvious that  $V_1^* = V^*$ .<sup>3</sup>

Notice that each optimal control sub-problem is well-behaved, we will not spend space on writing the corresponding standard Pontryagin conditions. Rather, we will focus on uncovering the much trickier optimality conditions with respect to the timing variables and the so-called matching conditions. Matching conditions refer to how hamiltonians and the co-state variables behave at the optimal junction times. This is solved by the following theorem.<sup>4</sup>

**Theorem 1.** *Let  $0 < t_K^* < t_P^* < \infty$  be the optimal timing. Then:*

$$(1) \quad H_2^*(t_P) = H_3^*(t_P) \text{ and } H_1^*(t_K) = H_2^*(t_K),$$

$$(2) \quad \lambda_K^{22*}(t_P) = \lambda_K^{21*}(t_P), \lambda_K^{21*}(t_K) = \lambda_K^{11*}(t_K) \text{ and } \lambda_P^{21*}(t_K) = \lambda_P^{11*}(t_K).$$

A few comments are in order here. First of all, one can read the five optimality conditions above are continuity or matching conditions at the junction times. In this respect, conditions (1) impose the continuity of the hamiltonian at the optimal junction times while the other conditions ensure the continuity of co-state variables at these times. Interestingly enough, one can observe that while at the technological switching time, both co-state variables are optimally continuous, only the one associated to  $K$  is necessarily continuous at the

ecological switching time. This points at the major difference between the two switching types: in the latter, pollution is fixed at the switching time, equal to the threshold value, while at the technological switching time, both state variables can be freely chosen. This generally implies discontinuity of the co-state variable associated to  $P$  at  $t_P$ .

Second, one can interpret the matching conditions (1)-(2) as first-order optimal timing conditions for  $t_P$  and  $t_K$  respectively. Generally speaking, the matching condition for timing  $t_i$  may be therefore written as:  $H_i^*(t_v) - H_{i+1}^*(t_v) = 0$ , for  $i = 1, 2$  and  $v = K, P$ . This condition is quite common in the literature of multi-stage technological switching (see Saglam 2011). We show here that it applies also to ecological switches. Keeping the discussion non-technical, one may interpret the difference  $H_i^*(t_v) - H_{i+1}^*(t_v)$  as the marginal gain from extending the regime inherent to the time interval  $[t_{i-1}, t_i]$ , with  $t_0 = 0$ , at the expense of the regime associated with interval  $[t_i, t_{i+1}]$ . Because there are no direct switching costs, the marginal switching cost is nil. Therefore, the matching conditions on hamiltonians do equalize marginal benefits and costs of delaying switching times. Hence they do feature first-order necessary conditions with respect to the latters.<sup>5</sup>

There are two remaining cases of interest with two interior switchings, which derive quite trivially from the analysis of the benchmark case. If ecological switching precedes technological switching, that is if  $0 < t_P < t_K < \infty$ , Theorem 1 still applies integrally. It is indeed invariant to the sign of  $t_K - t_P$  as one can infer from the discussion following Theorem 1. The last two (interior) switchings case follows the same logic though the list of corresponding first-order timing and matching conditions is shorter because only two successive regimes are involved, not three: one before  $t_K = t_P = t^s$  and one after. Denoting by  $H_i$ ,  $\lambda_K^{ii} = \lambda_K^i$  and  $\lambda_P^{ii} = \lambda_P^i$ ,  $i = 1, 2$ , the hamiltonians and co-state variables corresponding to the sub-problems on the intervals  $[0, t^s]$  and  $[t^s, \infty)$  respectively, optimality conditions reduce to  $H_1^*(t^s) = H_2^*(t^s)$  and  $\lambda_K^{1*}(t^s) = \lambda_K^{2*}(t^s)$ .

We end this section by assessing briefly the impact of technical and ecological switching on steady state. For any technological menu  $i = 1, 2$  and ecological regime  $j = 1, 2$ , the

expression of steady state capital and pollution are implicitly given by

$$U'((A_i - \delta)K_\infty^{ij}) = \frac{q_i D'(\frac{q_i K_\infty^{ij}}{\alpha_j})}{(A_i - \delta - \rho)(\rho + \alpha_j)} \text{ and } P_\infty^{ij} = \frac{q_i}{\alpha_j} K_\infty^{ij}.$$

One can directly observe that both  $K_\infty^{ij}$  and  $P_\infty^{ij}$  are non monotonic in the technological menu  $(A_i, q_i)$ . In addition, if  $K_\infty^{ij}$  is decreasing in  $\alpha_j$ ,  $P_\infty^{ij}$  also is not monotonic in  $\alpha_j$ . Intuition runs as follows. Suppose the planner adopts, at some instant, the new technological menu. The reduction in  $A_i$  generates the usual income and substitution effects that ambiguously affect investment. Moreover, the associated decrease in the intensity of pollution  $q_i$ , induced by the adoption of the new technology, lowers the social cost of capital accumulation and stimulates investment. So, there are different forces at play and it is not immediately obvious which effect will prevail in the long run. In the same vein, the impact of technology adoption on pollution is unclear. Of course, the decrease in  $q_i$  is a means to slow down pollution accumulation. But depending on whether technical change stimulates investment, there is an indirect effect that may go the other way round. In the numerical exercise to follow, we find it reasonable to assume that the adoption of the new cleaner technology allows the economy to reach a less polluted steady state. Regarding the impact of the ecological switch on steady state pollution, two effects are also pushing in opposite direction. Indeed, a decrease in  $\alpha_j$  makes it more difficult for Nature to regenerate itself. At the same time however, everything else equal, it tends to reduce the incentive to invest in capital (through the higher social cost of pollution  $\lambda_p^{ij}$ ). From now on, we shall consider the more realistic case where the ecological switch translates into higher pollution.

It is now time to apply this theoretical analysis and our solution approach to the optimal growth model described in Section 2.

### Numerical investigation

What should be the solution of the optimal growth problem with ecological and technical switches? To what extent does this solution respond to changes in the fundamentals of the

economy? What is the relationship between the two state variables, capital and pollution, along the optimal path? These are the questions we shall address in this numerical analysis.

### *Model calibration*

The analysis is conducted with the following utility and damage functions:  $U(C) = \theta C(\bar{C} - C)$  and  $D(P) = \nu P$ , with  $\theta, \nu, \bar{C} > 0$ . The set of baseline parameters used is

$$(3) \quad \begin{aligned} \theta &= 14.4, q_1 = 0.04, q_2 = 0.02, \bar{z} = 120, A_1 = 1/3, A_2 = 0.25, K_0 = 15, \\ P_0 &= 107, \rho = 0.05, \delta = 0.075, \alpha_1 = 0.005, \alpha_2 = 0.002, \bar{c} = 10, \nu = 2.1 \end{aligned}$$

As for the discount rate, the chosen value is close to what most western governments use for most long term investments. The depreciation rate is usually between 5% and 10% depending on the level of economic development. We choose 7.5%. A review of the literature suggests that  $\alpha_1 = 0.005$  (implying a half-life of 139 years, see Hoel and Karp 2002). The technology parameter value ( $A_1$ ) is set in order to be consistent with the observed  $K^{11}/Y^{11} = 3$ . Moreover in our model, emissions before technology adoption are  $E^{11} = q_1 K^{11}$ . According to the DICE model (Nordhaus 2008, figure 5-10 p.110),  $E^{11}/Y^{11} = 0.12$  for 2010, yielding  $q_1 = 0.04$ . At the steady state,  $\nu = -\lambda_{P^\infty}^{11}(\alpha_1 + \rho)$ . According to the literature (see for instance Nordhaus 2008), the social damage costs of carbon dioxide emissions cannot be than less than 20\$ or 30\$ per ton but could probably not exceed 50\$ per ton. Choosing 40\$ to appraise the shadow price of pollution we obtain  $\nu = 2.1$ . At the steady state again,  $\theta = \gamma \lambda_{P^\infty}^{11} q_1 / (\rho + \delta - A_1)$  where  $\gamma$  stands for the relative risk aversion. We assume the latter to be equal to 5 that leads to  $\theta = 14.4$ . Parameter  $\bar{C}$  in the utility function has been arbitrarily chosen because it is a scale parameter that does not affect conclusions of the simulations. Consistently with the discussion about the impact of ecological and technical switches on steady state, in the benchmark we choose parameters  $(A_2, q_2, \alpha_2)$  so that steady state pollution is lower (resp. larger) after the technical (resp. ecological) switch than before. For the values reported in (3), it appears that steady state capital is higher after the technical switch than before.<sup>6</sup> So, if the adoption of the new tech-

nology is costly in terms of investment in the short run, it is ultimately beneficial to capital accumulation.

The last important discussion refers to the set of initial conditions. For the particular issue of climate change, irreversible switches will likely occur if no severe action is undertaken soon. Indeed, there is now growing evidence that oceans (the most important carbon sink) display a buffering capacity that begins saturating. At the same time, the assimilation capacity of terrestrial ecosystems will likely peak by mid-century and then decline to become a net source of carbon by the end of the present century. Finally, the potential collapse of the North Atlantic meridional overturning circulation is drawing much of the attention, since it may happen for a 450 ppm  $CO_2$  concentration while we have already reached 390 ppm (Yohe et al. 2006). These considerations have led us to start the numerical exercise by setting the level of initial pollution close enough to – but below – the ecological threshold. In addition, it is worth noting that we have chosen a threshold  $\bar{P} < P_\infty^{11}$ . It implies that a switch, either technical and/or ecological, will necessarily occur at some point in time. We further assume that a technological switch is a priori a means to avoid the ecological switch because  $\bar{P} > P_\infty^{21}$ . Finally, the initial endowment in capital  $K_0$  satisfies  $K_0 < K_\infty^{11} (< K_\infty^{21})$ .

### *Benchmark scenario*

Solving our calibrated AK model with pollution and switches, we first observe the existence of multiple optimality candidates. For the set of baseline parameters, we find solutions to the necessary optimality conditions associated with three of the nine possible regimes: 1. Technological switch alone, 2. Simultaneous technological and ecological switches and 3. Immediate technological switch alone.

Interestingly enough, as far as the nature of the relationship between  $K$  and  $P$  is concerned, these three candidates show very distinct features. Along the first regime, there is a sustained capital accumulation during the period before the switch, investment being more efficient than after  $t_K$ . Consequently,  $K_\infty^{21}$  is nearly reached at the time of the switch and pol-

lution rises as well during this phase (see figure 1, top). At some point in time ( $t_K = 15.61$ ), the accumulated pollution and the level of capital become so high that the economy finds it worthwhile to adopt the new greener technology. From that point on, pollution, starting at a high level, decreases because natural assimilation now prevails on the lesser emissions due to the new technology. In the second regime, capital accumulation is delayed relative to what happens in the first regime in order to reach  $\bar{P}$  as late as possible (see figure 1, second row). Once the double switch has occurred, pollution grows at a high rate because the economy cannot rely anymore on a high regeneration rate. Finally, the economy may choose to adopt immediately the new technology. In this third regime, we obtain a U-shape relationship between  $K$  and  $P$  (see Figure 1, third row): from the beginning, the economy is able with the new technology to accumulate capital while reducing the pollution stock.

What is the optimal regime? It turns out that it is the regime with a technical switch alone ( $W_1 = 2464.6 > W_2 = 2432.81 > W_3 = 1948.3$ ). This result is very intuitive. Along regime 1, the planner can avoid the ecological threshold and is therefore better off than under regime 2. Regime 1 also dominates regime 3 because under the former, capital accumulation benefits from a higher investment efficiency. To conclude this analysis, it is worth mentioning that the optimal policy, with technical change alone, exhibits a capital-pollution relationship that has the feature of the EKC.<sup>7</sup>

Let us investigate whether these results survive to modifications of critical parameters.

### *Sensitivity analysis*

This section reviews all the possible variations around the baseline scenario. Table 1 summarizes our findings.

Several interesting conclusions hold whatever the scenario considered. Firstly, the regime with immediate technical switch always exists and is always dominated by another solution. Secondly, there is no solution featuring a technical switch followed by an ecological switch. Thirdly, each time a candidate with a technical switch alone exists, this

yields the optimum. Last but not least, when the optimum is a technological switch alone, the relationship between capital exhibits a turning point and consequently is inverted U-shaped provided that initial capital and pollution are high enough<sup>8</sup> and that agents are sufficiently sensitive to pollution.

Finally, it is worth mentioning that under some scenarios, our conclusions sensibly differ. In case where the intensity of pollution remains relatively high after adoption ( $q_2 = 0.03$ ), the technological switch is not valuable to the planner. It implies that the optimum is the regime with a simultaneous double switch. For a not too damaging pollution ( $v = 1$ ), a small ecological threat ( $\alpha_2 = 0.003$ ), or more impatient agents ( $\rho = 0.1$ ), less attention is paid to the ecological threshold and solutions to the regime with an ecological switch alone exist. However, it is optimal only in the case where agents are relatively impatient. In such a case, the capital-pollution relationship is not at all an EKC: it encompasses an intermediate stage where pollution increases whereas capital decreases (see figure 1, bottom).

### **Concluding remarks**

This paper investigates the income-pollution relationship within an optimal AK growth model with technological and ecological switches. We show that the EKC, that is usually seen as a description of the relationship between wealth and pollution along the different development stages of one country, can also emerge as a result of the implementation of the optimal policy from the current development stage of the economy.

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## Notes

<sup>1</sup>The case  $\alpha_2 = 0$  is the case for strong pollution irreversibility, we allow here for intermediate situations where the decay goes down above the threshold while nature keeps some degree of regeneration.

<sup>2</sup>The case of two switchings, one interior and one corner, can be also settled similarly. For example if  $0 < t_P < \infty$  and  $t_K = 0$ , one have just to solve for the ecological switch the particular problem with  $t_K = 0$  given and the implied capital law of motion.

<sup>3</sup>One would use exactly the same scheme to handle a dynamic optimization problem in discrete time over three periods. Here the Bellman principle applies on the three intervals involved by the double timing problem instead of discrete periods of time.

<sup>4</sup>A detailed proof of the theorem is available upon request.

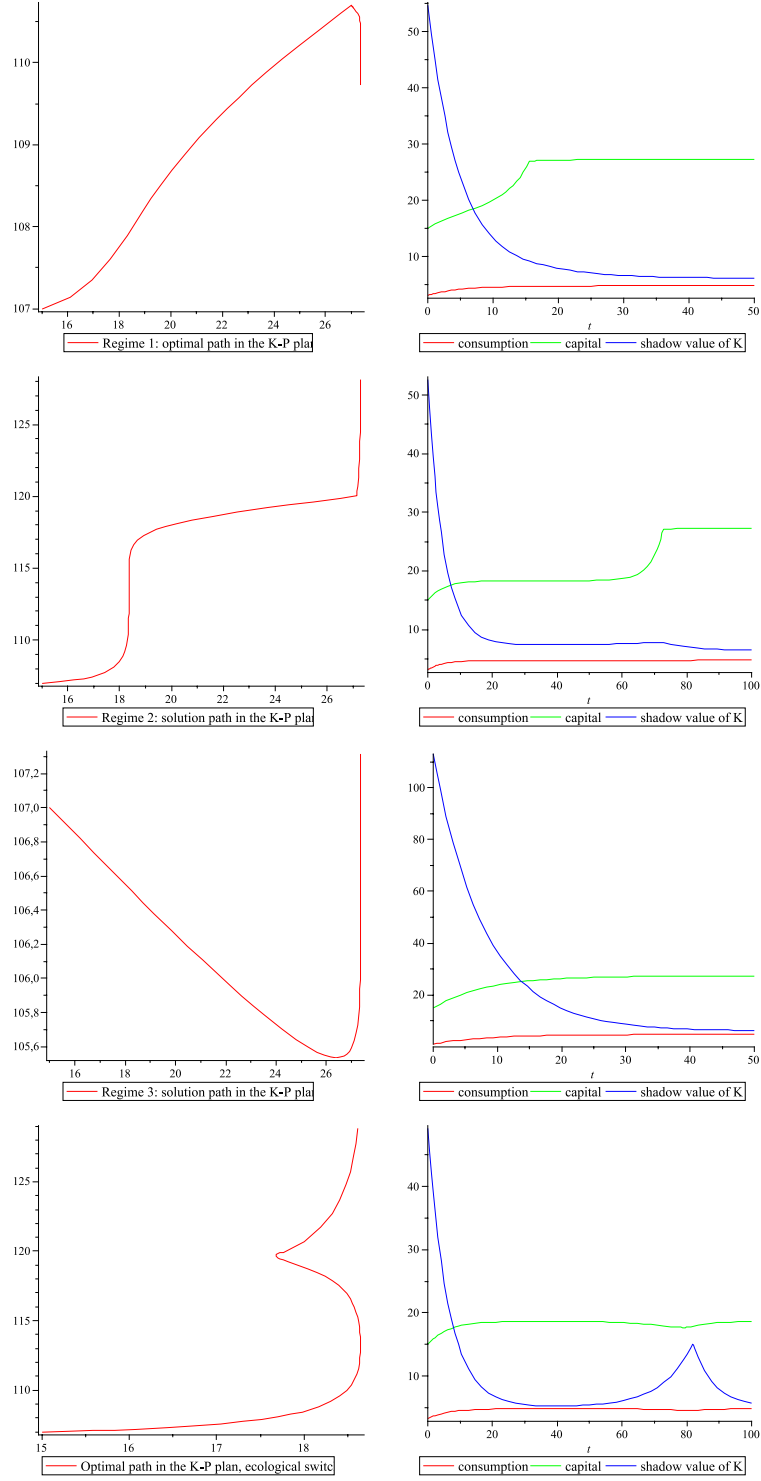
<sup>5</sup>The same type of arguments could be used to visualize easily the kind of necessary conditions implied by corner switching times: for example immediate technological switching, say  $t_K^* = 0$  implies  $H_1^*(0) - H_2^*(0) < 0$ .

<sup>6</sup> Steady state values, for the benchmark, are:  $(P_\infty^{11}, K_\infty^{11}) = (147, 18.4)$ ,  $(P_\infty^{21}, K_\infty^{21}) = (109.4, 27.4)$ ,  $(P_\infty^{12}, K_\infty^{12}) = (366.3, 18.3)$  and  $(P_\infty^{22}, K_\infty^{22}) = (272.9, 27.3)$ .

<sup>7</sup> Would  $A_2$  be closer to  $A_1$  (for instance  $A_2 = 0.3$ , see table 1) the investment efficiency gap before and after the switch would be reduced and the capital would be significantly increasing after the switch, therefore leading to a nicer EKC.

<sup>8</sup> For a low  $K_0$ , assimilation prevails on emission at the beginning, thus generating a decreasing relationship between  $K$  and  $P$ . For a low  $P_0$ , the pollution keeps raising even after the adoption of the new technology.

## Figures



**Figure 1. Benchmark scenario: optimality candidates and corresponding paths. Last row: optimum with ecological switch ( $\rho = 0.1$ )**

## Tables

**Table 1. Summary of results**

	Tech. switch	Eco. switch	$t_K < t_P$	$t_P < t_K$
Bench.	$t_K = 15.6, V = 2464.6$	no	no	no
$K_0 = 5$	$t_K = 22, V = 1257.9$	no	no	no
$P_0 = 50$	$t_K = 15.6, V = 4641$	no	no	no
$K_0 = 5, P_0 = 50$	$t_K = 22, V = 3434, 3$	no	no	no
$q_2 = 0.01$	$t_K = 10.2, V = 2578.2$	no	no	no
$q_2 = 0.03$	no	no	no	no
$\alpha_2 = 0.001$	$t_K = 15.6, V = 2464.6$	no	no	no
$\alpha_2 = 0.003$	$t_K = 15.6, V = 2464.6$	$t_P = 96, V = 2431, 3$	no	$(t_P, t_K) = (108.4, 122.2), V = 2431.5$
$\nu = 1$	$t_K = 19.7, V = 4872.8$	$t_P = 78.6, V = 4857.4$	no	no
$\rho = 0.1$	no	$t_P = 81.9, V = 1206.5$	no	no
$A_2 = 0.3$	$t_K = 5.9, V = 2568$	no	no	no
	$t_K = 0$	$t_K = t_P = t$	Opt.	Relation $K, P$
Bench.	$V = 1948.3$	$t = 72.7, V = 2428.4$	Tech. switch	EKC
$K_0 = 5$	$V = 378.4$	no	Tech. switch	no EKC
$P_0 = 50$	$V = 4124.7$	$t = 247.4, V = 4610.2$	Tech. switch	no EKC
$K_0 = 5, P_0 = 50$	$V = 2563.8$	no	Tech. switch	no EKC
$q_2 = 0.01$	$V = 2132$	$t = 72.5, V = 2434.2$	Tech. switch	EKC
$q_2 = 0.03$	$V = 1768$	$t = 72.9, V = 2422.7$	switch $t_K = t_P$	no EKC
$\alpha_2 = 0.001$	$V = 1948.3$	$t = 74, V = 2426.1$	Tech. switch	EKC
$\alpha_2 = 0.003$	$V = 1948.3$	$t = 71.5, V = 2430.8$	Tech. switch	EKC
$\nu = 1$	$V = 4280.8$	$t = 64.8, V = 4856.8$	Tech. switch	EKC
$\rho = 0.1$	$V = 714.58$	$t = 68.8, V = 1206.4$	Tech. switch	no EKC
$A_2 = 0.3$	$V = 2503.2$	$t = 84.8, V = 2432.6$	Tech. switch	EKC